LLM Magnons

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Introduction

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- AdS/CFT conjecture - IIB string theory on $AdS_5 \times S_5$ dual to $\mathcal{N} = 4$ SYM defined on the AdS boundary
- String theory Hamiltonian (energies) $\Leftrightarrow$ gauge theory dilatation operator (scaling dimension)
- Convention: N the gauge group dimension. Think of the fields as $N \times N$ matrices

AdS/CFT conjecture

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- Dual gauge theory: Operators consisting of $O(1), O(\sqrt{N})$ fields
- Loop operators: $O(\{n_l\}) = Tr\left(Z^{n_1}YZ^{n_2}...YZ^{n_k}Y\right) = Y^{i_1}_{i_{\sigma(1)}}...Y^{i_m}_{i_{\sigma(m)}}Z^{i_{m+1}}_{i_{\sigma(m+1)}}...Z^{i_{m+n}}_{i_{\sigma(m+n)}}$

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- Very useful mapping to spin chains
- Orthogonal basis in the large N limit

AdS/CFT conjecture

- For larger operators e.g. $O(N)$ the loop operators are no longer orthogonal
- Schur polynomials: $\chi_T(Z) = \frac{1}{n!}\sum_{\sigma\in S_n}\chi_T(\sigma) Z^{i_1}_{i_{\sigma(1)}}\ldots Z^{i_n}_{i_{\sigma(n)}}$
- $O(N)$ operators dual to giant gravitons - $D3$ branes wrapping an S^3 of AdS_5 or an S^3 of S^5
- Schur polynomials of $O(1)$ long rows or $O(1)$ long columns
- The backreaction to the $AdS_5 \times S^5$ geometry can be dropped

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- $ds^2 = -h^{-2}(dt + V_i dx^i)^2 + h^2(dy^2 + dx^i dx^i) + ye^G d\Omega_3^2 + ye^{-G} d\tilde{\Omega}_3^2$
- $z = \frac{1}{2}\tanh(G) \quad h^{-2} = 2y\cosh(G) \quad$ et cetera
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- Concentric circles: Dual to Schur polynomials with $O(N)$ rows and $O(N)$ columns

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- String excitations represented as directed line segments on the LLM plane representing energy and momentum
- Can they be reproduced on the gauge theory side for LLM?
- One loop: $D = -\frac{g_{YM}^2}{8\pi^2}Tr\left([Y,Z]\left[\frac{d}{dY},\frac{d}{dZ}\right]\right)$

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- $O(\{n_l\}) = Tr\left(Z^{n_1} YZ^{n_2} \dots YZ^{n_k} Y\right) = Y^{i_1}_{i_{\sigma(1)}} \dots Y^{i_m}_{i_{\sigma(m)}} Z^{i_{m+1}}_{i_{\sigma(m+1)}} \dots Z^{i_{m+n}}_{i_{\sigma(m+n)}}$
- $O(\{n\}) = \sum_{T,(t,u)\alpha\beta} \frac{d_T n! m!}{d_t d_u (n+m)!} \chi_{T,(t,u)\alpha\beta}(\sigma_{\{n\}}) \chi_{T,(t,u)\alpha\beta}(Z, Y)$
- u has m boxes, t has n boxes and T has $m+n$ boxes

Localised operators

- Proposal: $O_B(\{n_k\}, Z, Y) = \sum_{T,(t,u)\alpha\beta} \sqrt{\frac{f_{T_B} \text{hooks}_{t_B} \text{hooks}_u}{\text{hooks}_{T_B}}} \chi_{T_B,(t_B,u)\alpha\beta}(\sigma^{-1}_{\{n_k\}}) O_{T_B,(t_B,u)\beta\alpha}(Z, Y)$
- Note that we are only summing over T, t and u
- t_B organises all the Z boxes in the operator

Simplifying the hooks

- First simplification
- $\frac{\text{hooks}_{t_B}\text{hooks}_u}{\text{hooks}_{T_B}} = \kappa^m \frac{\text{hooks}_t\text{hooks}_u}{\text{hooks}_T}(1 + O(N^{-1}))$
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- Young's orthogonal representation
- $(i, i+1)|\text{pattern}\rangle = \frac{1}{c_i - c_{i+1}}|\text{pattern}\rangle + \sqrt{1 - \frac{1}{(c_i - c_{i+1})^2}}|\text{swapped pattern}\rangle$
- Boxes that are well separated always swap

Simplifying the restricted characters

- $\chi_T(\sigma) = \sum_{\text{patterns}} \langle \text{pattern} | \sigma | \text{pattern} \rangle \quad \text{e.g.} \quad \chi_T((1)) = d_T.$

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- When boxes are well separated they always swap $\Rightarrow$ restricted character vanishes
- $\chi_{T_B,(t_B,u)\alpha\beta}(\sigma^{-1}_{\{n_k\}}) = d_B \chi_{T,(t,u)\alpha\beta}(\sigma^{-1}_{\{n_k\}})$

Action of the Dilitation Operator

- Proposal: $O_B(\{n_k\}, Z, Y) = \sum_{T,(t,u)\alpha\beta} \sqrt{\frac{f_{T_B} \text{hooks}_{t_B} \text{hooks}_u}{\text{hooks}_{T_B}}} \chi_{T_B,(t_B,u)\alpha\beta}(\sigma^{-1}_{\{n_k\}}) O_{T_B,(t_B,u)\beta\alpha}(Z, Y)$
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- A bit more detail: $DO_{R_B,(r_B,s)\mu_1\mu_2}(Z,Y) = \sum_{T,(t,u)\nu_1\nu_2} N_{R_B,(r_B,s)\mu_1,\mu_2;T,(t,u)\nu_1\nu_2} \times O_{T,(t,u)\nu_1\nu_2}(\mathrm{Z},\mathrm{Y})$
- $N_{R_B,(r_B,s)\mu_1,\mu_2;T,(t,u)\nu_1\nu_2} \sim Tr\left(\left[(1,m+1), P_{R_B,(r_B,s)\mu_1\mu_2}\right] I_{R'_B,T'} \left[(1,m+1), P_{T,(t,u)\nu_2\nu_1}\right] I_{T',R'_B}\right)$
- The intertwiner I_{T',R'_B} is only non-zero when T and R_B differs by the placement of a single box

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- Y box put in a distant spot - the permutation $(1, m+1)$ leaves the boxes inert $(\frac{1}{N})$
- (Rough Argument). Distant Z box can either swap in or out of closed string state. Swapping out (in) far $\rightarrow$ character is zero

More precisely

- $$DO_B(\{n\}) = \sum_{p=1}^{m} \sum_{q=m+1}^{m+n} \sum_{T,t,u,\nu_1\nu_2} \sum_{T^+} N_{T_B,t_b,u} \times \times \chi_{T^+,(t,u,)\nu_1\nu_2}(\psi^{-1}) \times O_{T_B,(t_B,u)\nu_2\nu_1}(Z,Y)$$
- ψ is a permutation consisting of four terms conjugate to $\sigma_n(p,q)(m+n+1,q)$, $(m+n+1,q)\sigma_n(p,q)$, $(m+n+1,q)\sigma_n(p,q)$, $(m+n+1,q)(p,q)\sigma_n$
- $(p,q)\sigma_n$ and $\sigma_n(p,q)$ is the product of a $m+n-k$ and a k cycle

Putting it all together

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Conclusion

- The restricted Schur language is well suited to dealing with excitations even of LLM geometries
- Semi-classical physics can emerge in the large N limit, even if the ("background") operators are $O(N^2)$
- Gauge theory mechanism for distinguishing background and excitation
- ***Large N Integrability still holds as long as the excitations are attached to the same corners

Outlook

- Could the $O(N)$ corner diagrams be understood?
- Understanding the outward pointing corners
- Excitations stretching between multiple corners
- Can a constructive procedure be found that produces the appropriate metric from a gauge theory operator?

Thank you for your attention!